Short Course I:
SINGULAR as a Tool for Singularity Theory

Suggestions for the Hands-On Tutorial

In the following, you will find several tasks to be solved inside SINGULAR. We have "classified" them by *'s to indicate the difficulty.

SOME HINTS:

- If you are looking for a command, use the help system by typing `help`⟨topic⟩`;`.
- If you are not sure what the name of the command might be, type `help index;` to look in the index of the online manual or look in the help page of the corresponding data type for related commands.

(1*) Compute in SINGULAR:

- $3 + 5$,
- $\frac{3}{5} + \frac{8}{9}$.

(2*) Define the `matrix`

$$M = \begin{pmatrix} a & b & c \\ z & y & x^3 \\ x & z & y \end{pmatrix} \subset \mathrm{Mat}_{3\times 3}(\mathbb{Q}[a,b,c,x,y,z]),$$

and check whether it was defined correctly by `print`ing it to the screen.

Compute the `det`erminant of M.

Determine the `coef`ficients (in $\mathbb{Q}[b,c,x,y,z]$) of this determinant, considered as element of $\big(\mathbb{Q}[b,c,x,y,z]\big)[a]$.

(3**) Is the `poly`nomial $x^2 + y^3$ contained in the `ideal` generated by $x + xy^3, y^2 + y^4$

- in the `ring` $\mathbb{Q}[x,y]$?
- And in the `ring` $\mathbb{Q}[x,y]_{<x,y>}$?
- And in $\mathbb{Q}(y)[x]$?

HINT: you can use the `reduce` command.

(4)** In the ring $\mathbb{Q}[x,y]_{<x,y>}$, define the polynomial $f = x^3 + y^5$. Compute the $\mathbb{Q}$-vector space dimension of the ring

$$\mathbb{Q}[x,y]_{\langle x,y\rangle} \Big/ \left\langle f, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle .$$

Find a monomial $\mathbb{Q}$-vector space basis for it.

The number which was just computed is the *Tjurina number* of the plane curve singularity defined by f. Also compute this number using the corresponding SINGULAR-library command.

(5)** In the ring $R = (\mathbb{Z}/7\mathbb{Z})[x,y,z]$ define two polynomials f and g which are random linear combinations of the variables.

Compute the dimension of the quotient ring $R/\langle f, g\rangle$.

(6)** Find equations in $\mathbb{Q}[x,y,z]$ defining the curve given by $t \longmapsto (t^3, t^5, t^7)$.

Is the ideal I generated by these equations a radical ideal?

Compute a minimal free resolution of $\mathbb{Q}[x,y,z]/I$, and display the corresponding maps.

(7*)** Compute all singular points of the affine complex plane curve defined by

$$(x^2 + y^2 - 1)^3 + 27x^2y^2 = 0$$

and determine the respective type of singularity.

(8*)** Write a SINGULAR procedure, which computes the inverse of a polynomial up to a given order (provided that the polynomial is a unit in the current ring).